(1)

$$\int \frac{1}{\sqrt{x}\,(16-x^2)}\,dx$$

$$I_1 = \left| \begin{array}{l} t=\sqrt{x} \\ x=t^2 \\ dx = 2t\,dt \end{array} \right| = \int \frac{1}{t\,(16-t^4)} \cdot 2t\,dt$$

$$= \int \frac{2}{16-t^4}\,dt = \int \frac{-2}{(t^2-2)(4+t^2)}\,dt$$

$$= \int \frac{-2}{(t-2)(t+2)(4+t^2)}\,dt = \frac{A}{t-2} + \frac{B}{t+2} + \frac{Ct+D}{4+t^2}$$

$$A = \frac{-2}{(2+2)(4+4)} = -\frac{1}{16} \qquad B = \frac{1}{16}$$

$$-2 = A(t+2)(4+t^2) + B(t-2)(4+t^2) + (Ct+D)(t^2-4)$$

$$-2 = t^3(A+B+C) + t^2(2A-2B+D)$$

tedy

$$A+B+C=0 \qquad C=0$$

$$2A-2B+D=0 \qquad D=2B-2A=\frac{1}{4}$$

$$\int \frac{1}{t-2}\,dt \overset{c}{=} \log|t-2| \qquad \int \frac{1}{t+2}\,dt \overset{c}{=} \log|t+2|$$

$$\int \frac{1}{t^2+4}\,dt = \frac{1}{4}\int \frac{1}{\left(\frac{t}{2}\right)^2+1}\,dt \overset{c}{=} \frac{1}{2}\,\mathrm{arctg}\left(\frac{t}{2}\right)$$

Tedy

$$\int \frac{2}{16-t^4}\,dt \overset{c}{=} -\frac{1}{16}\log|t-2| + \frac{1}{16}\log|t+2| + \frac{1}{8}\,\mathrm{arctg}\left(\frac{t}{2}\right)$$

a

$$\int \frac{2}{\sqrt{x}\,(16-x^2)}\,dx \overset{c}{=} -\frac{1}{16}\log|\sqrt{x}-2| + \frac{1}{16}\log|\sqrt{x}+2| + \frac{1}{8}\,\mathrm{arctg}\left(\frac{\sqrt{x}}{2}\right)$$

maximální intervaly $(0,4)$, $(4,\infty)$

(2)

(a) $$T^{6}_{0,\cos x} = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{6!}$$

$$T^{6}_{0,\cos x} = 1 - \frac{x^4}{2}$$

$$T^{6}_{0,x^2\cos x} = x^2 - \frac{x^4}{2} + \frac{x^6}{24}$$

(b) $$T^{6}_{0,\cos(x^2\cos x)} = 1 - \frac{1}{2}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{24}\right)^2$$

$$+ \frac{1}{24}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{24}\right)^4 - \frac{1}{6!}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{24}\right)^6 + o(x^6)$$

tyto členy bych ani psát nemusel

$$= 1 - \frac{x^4}{2} + x^6\left(2\cdot\left(-\frac{1}{2}\right)\cdot\left(-\frac{1}{2}\right)\right)$$

$$= 1 - \frac{x^4}{2} + \frac{x^6}{2}$$

(c) $$\lim_{x\to 0} \frac{\cos(x^2\cos x) - \cos(x^2)}{x^6} = \lim_{x\to 0} \frac{\frac{x^6}{2} + o(x^6)}{x^2} = \frac{1}{2}$$

tedy $k=6$ a $C=\frac{1}{2}$

(taková k a C jsou určena jednoznačně,

pokud $\cos(x^2\cos x)-\cos(x^2) \sim C\cdot x^k$, $x\to 0$,

a $\cos(x^2\cos x)-\cos(x^2) \sim D\cdot x^n$, $x\to 0$,

potom i $C\cdot x^k \sim D\cdot x^n$, $x\to 0$,

čili $1=\lim\limits_{x\to 0}\dfrac{Cx^k}{Dx^n}=\dfrac{C}{D}\cdot\lim\limits_{x\to 0} x^{k-n}$,

ale to je možné pouze pro $k=n$ a $C=D$.)